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Modeling and Optimization of the Compressive Strength of Latertic Concrete Using Scheffe's Theory

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Abstract: - The paper presents the report of an investigation carried out to model and optimize the compressive strength of Lateritic Concrete. The laterite is the reddish soil layer often belying the top soil in many locations and further deeper in some areas, collected from the Vocational Education Building Site of the University of Nigeria, Nsukka. The work applied the Scheffe's optimization approach to obtain a mathematical model of the form $f(x_{i1},x_{i2},x_{i3})$, where x_i are proportions of the concrete components, viz: cement, laterite and water. Scheffe's experimental design techniques are followed to mould various block samples measuring 220mm x 210mm x 120mm, with varying generated components ratios which were tested for 28 days strength. To carry out the task, we embark on experimentation and design, applying the second order polynomial characterization process of the simplex lattice method. The model adequacy is checked using the control factors. Finally a software is prepared to handle the design computation process to select the optimized properties of the mix, and generate the optimal mix ratios for the desired property.

Keywords: *Concrete, laterite, pseudo-component, Simplex-lattice, model.*

I. INTRODUCTION

The construction of structures is a regular operation which creates the opportunity for continued change and improvement on the face of the environment. From the beginning of time, the cost of this change has been of major concern to man as the major construction factors are finance, labour, materials and equipment.

Major achievements in the area of environmental development is heavily dependent on the availability of construction materials which take a high proportion of the cost of the structure. This means that the locality of the material and the usability of the available materials directly impact on the achievable development of the area as well as the attainable level of technology in the area.

In the present time, concrete is the main material of construction, and the ease or cost of its production accounts for the level of success in the of area environmental upgrading through the construction of new roads, buildings, dams, water structures and the renovation of such structures. To produce the concrete several primary components such as cement, sand, gravel and some admixtures are to be present in varying quantities and qualities. Unfortunately, the occurrence and availability of these components vary very randomly with location and hence the attendant problems of either excessive or scarce quantities of the different materials occurring in different areas. Where the scarcity of one component prevails exceedingly, the cost of the concrete production increases geometrically. Such problems obviate the need to seek alternative materials for partial or full replacement of the component when it is possible to do so without losing the quality of the concrete.

1.1 Optimization Concept

Every activity that must be successful in human endeavour requires planning. The target of planning is the maximization of the desired outcome of the venture. In order to maximize gains or outputs it is often necessary to keep inputs or investments at a minimum at the production level. The process involved in this planning activity of minimization and maximization is referred to as optimization, (Orie O.U. and Osadebe N.N., 2009). In the science of optimization, the desired property or quantity to be optimized is referred to as the objective function. The raw materials or quantities whose amount of combinations will produce this objective function are referred to as variables.

The variations of these variables produce different combinations and have different outputs. Often the space of variability of the variables is not universal as some conditions limit them. These conditions are called constraints. For example, money is a factor of production and is known to be limited in supply. The constraint at any time is the amount of money available to the entrepreneur at the time of investment.

Hence or otherwise, an optimization process is one that seeks for the maximum or minimum value and at the same time satisfying a number of other imposed requirements (Majid, K.I., 1974). The function is called the objective function and the specified requirements are known as the constraints of the problem.

Everybody can make concrete but not everybody can make structural concrete. Structural concrete are made with specified materials for specified strength. Concrete is heterogeneous as it comprises sub-materials.

Concrete is made up of fine aggregates, coarse aggregates, cement, water, and sometimes admixtures. David and Galliford (2000), report that modern research in concrete seeks to provide greater understanding of its constituent materials and possibilities of improving its qualities. For instance, Portland cement has been partially replaced with ground granulated blast furnace slag (GGBS), a by–product of the steel industry that has valuable cementations properties (Ecocem Ireland Ltd, 1993).

1.2 Concrete Mix optimization

The task of concrete mix optimization implies selecting the most suitable concrete aggregates from the data base (Genadij and Juris, 1998). Several methods have been applied. Examples are by Mohan (2002), Simon (2003), Lech (1999), Czarneki (1994). Nordstrom and Munoz (1994) proposed an approach which adopts the equilibrium mineral assemblage concept of geochemical thermodynamics as a basis for establishing mix proportions. Bloom and Bentur (1995) reports that optimization of mix designs require detailed knowledge of concrete properties. Low water-cement ratios lead to increased strength but will negatively lead to an accelerated and higher shrinkage. Apart from the larger deformations, the acceleration of dehydration and strength gain will cause cracking at early ages.

1.3**Modeling**

Modeling means setting up mathematical models/formulations of physical or other systems. Many factors of different effects occur in nature in the world simultaneously dependently or independently. When they interplay they could inter-affect one another differently at equal, direct, combined or partially combined rates variationally, to generate varied natural constants in the form of coefficients and/or exponents. The challenging problem is to understand and asses these distinctive constants by which the interplaying factors underscore some unique natural phenomenon towards which their natures tend, in a single, double or multi phase system.
For such assessment a model could be constructed for a proper observation of response from the interaction of the factors through controlled experimentation followed by schematic design where such simplex lattice approach of the type of Henry Scheffe (1958) optimization theory could be employed. Also entirely different physical systems may correspond to the same mathematical model so that they can be solved by the same methods. This is an impressive demonstration of the unifying power of mathematics (Erwin Kreyszig, 2004).

II. LITERATURE REVIEW

- To be a good structural material, the material should be homogeneous and isotropic. The Portland cement, laterite or concrete are none of these, nevertheless they are popular construction materials (Wilby, 1963).
- … laterized concrete can be used in constructing cylindrical storage structures (Ukamaka N.T., 2007).
- With given proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio (Reynolds, C. and Steedman, J.C, 1981).
- Tropical weathering (laterization) is a prolonged process of chemical weathering which produces a wide variety in the thickness, grade, chemistry and ore mineralogy of the resulting soils (Tardy, Yves - 1997).
- The mineralogical and chemical compositions of laterites are dependent on their parent rocks (Tardy Yves, 1997).
- Laterite formation is favoured in low topographical reliefs of gentle crests and plateaus which prevent the erosion of the surface cover (Dalvi, Ashok D.; Bacon, W. Gordon; Osborne, Robert C. (March 7–10, 2004)).
- Laterites reflect past weathering conditions (Helgren, David M.; Butzer, Karl W. Butzer, October 1977).
- Present-day laterite occurring outside the humid tropics are considered to be indicators of climatic change, continental drift. The mineralogical and chemical compositions of laterites are dependent on their parent rocks (Tardy Yves, 1997), a combination of both (Bourman, R.P. August 1993).
- Of all the desirable properties of hardened concrete such as the tensile, compressive, flexural, bond, shear strengths, etc., the compressive strength is the most convenient to measure and is used as the criterion for the overall quality of the hardened concrete (Majid, K.I., 1974).
- Every activity that must be successful in human endeavour requires planning whose target is the maximization of the desired outcome of the venture. (Orie O.U. and Osadebe N.N., 2009).
- Optimization process is one that seeks for the maximum or minimum value and at the same time satisfying a number of other imposed requirements (Majid, K.I., 1974).
- Modern research in concrete seeks to provide greater understanding of its constituent materials and possibilities of improving its qualities (David and Galliford, 2000).
- The task of concrete mix optimization implies selecting the most suitable concrete aggregates from the data base (Genadij and Juris, 1998).
- Optimization of mix designs require detailed knowledge of concrete properties (Bloom and Bentur, 1995).
- The task of concrete mix optimization implies selecting the most suitable concrete aggregates from a data base (Genadji and Juris, 1998).

- Mathematical models have been used to optimize some mechanical properties of concrete made from Rice Husk Ash (RHA), - a pozolanic waste (Scheffe 1958, Obam and Osadebe's, 2007) .
- The inclusion of mound soil in mortar matrix resulted in a compressive strength value of up to 40.08N/mm2, and the addition of 5% of mound soil to a concrete mix of 1:2:4:0.56 (cement: sand: coarse aggregate: water) resulted in an increase of up to 20.35% in compressive strength, (Felix et al, Alu and Sulaiman, 2000).
- Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture, and they are equidistant from each other (Jackson N., 1983).
- When studying the properties of a q-component mixture, which are dependent on the component ratio only the factor space is a regular (q-1)–simplex (S. Akhnazarov and V. Kafarov , 1982).
- Simplex lattice designs are saturated, that is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used (S. Akhnazarov and V. Kafarov, 1982).

III. BACKGROUND (SCHEFFE'S OPTIMIZATION) THEORY

This is a theory where a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. In the theory only a single phase mixture is covered. The theory lends path to a unifying equation model capable of taking varying componental ratios to fix approximately equal mixture properties. The optimization is the selectability, from some criterial (mainly economic) view point, the optimal ratio from the component ratios list that can be automatedly generated. His theory is one of the adaptations to this work in the formulation of response function for compressive strength of lateritic concrete.

3.1 Simplex Lattice

Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture (Jackson N., 1983), and they are equidistant from each other. Mathematically, a simplex lattice is a space of constituent variables of X_1, X_2, X_3,……, and X_i which obey these laws:

$$\left.\begin{array}{l} X_i < 0 \\ X \neq \text{negative} \\ 0 \leq x_i \leq 1 \\ \sum_{i=1} x_i = 1 \end{array}\right\} \quad \ldots\ldots 3.1$$

That is, a lattice is an abstract space.

To achieve the desired strength of concrete, one of the essential factors lies on the adequate proportioning of ingredients needed to make the concrete. Henry Scheffe, (1958), developed a model whereby if the compressive strength desired is specified, possible combinations of needed ingredients to achieve the compressive strength can easily be predicted by the aid of computer, and if proportions are specified the compressive strength can easily be predicted.

3.2 Simplex Lattice Method

In designing experiment to attack mixture problems involving component property diagrams the property studied is assumed to be a continuous function of certain arguments and with a sufficient accuracy it can be approximated with a polynomial (Akhnazarova and Kafarov, 1982, pp 242). When investigating multi-components systems the use of experimental design methodologies substantially reduces the volume of an experimental effort. Further, this obviates the need for a special representation of complex surface, as the wanted properties can be derived from equations while the possibility to graphically interpret the result is retained.

As a rule the response surfaces in multi-component systems are very intricate. To describe such surfaces adequately, high degree polynomials are required, and hence a great many experimental trials. A polynomial of degree n in q variable has C^n_{q+n} coefficients. If a mixture has a total of q components and x_1 be the proportion of the i^{th} component in the mixture such that,

$$x_i >= 0 \ (i=1,2, \ldots.q), \qquad (3.2)$$

then the sum of the component proportion is a whole unity i.e.

$X_1 + x_2 + x_3 = 1$ or $\sum x_i - 1 = 0$ …. .. .. .. (3.3)

where i = 1, 2, …., q… Thus the factor space is a regular (q-1) dimensional simplex. In (q-1) dimensional simplex if q = 2, we have 2 points of connectivity. This gives a straight line simplex lattice. If q=3, we have a triangular simplex lattice and for q = 4, it is a tetrahedron simplex lattice, etc. Taking a whole factor space in the design we have a (q,m) simplex lattice whose properties are defined as follows:

i. The factor space has uniformly distributed points,

ii. Simplex lattice designs are saturated (Akhnarova and Kafarov, 1982). That is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used.

Hence, for the quadratic lattice (q,2), approximating the response surface with the second degree polynomials (m=2), the following levels of every factor must be used 0, ½ and 1; for the cubic (m=3) polynomials, the levels are 0, 1/3, 2/3 and 1, etc; Scheffe, (1958), showed that the number of points in a (q,m) lattice is given by

$$C_{q+m-1} = q(q+1) \ldots (q+m-1)/m! \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad .. \quad .. \quad .. \quad .. \ (3.4)$$

3.2.1The (3,2) Lattice Model

The properties studied in the assumed polynomial are real-valued functions on the simplex and are termed responses. The mixture properties were described using polynomials assuming a polynomial function of degree m in the q-variable x_1, x_2 ……, x_q, subject to equation 2.3, and will be called a (q,m) polynomial having a general form:

$$\hat{Y} = \underset{i\le l\le q}{b_0} + \underset{i\le l<j\le q}{\sum b_i X_i} + \sum b_{ij} X_i \underset{i\le l<j<k\le q}{X_{ij}} + \ldots + \sum b_{ijk} + \sum b_{i1i2\ldots in} X_{i1} X_{i2} \ldots X_{in} \quad \ldots\ldots \quad .. \quad .. \ (3.5)$$

where b is a constant coefficient.

The relationship $\sum xi = 1$ enables the q^{th} component to be eliminated and the number of coefficients reduced to C^m_{q+m-1}, but the very character of the problem dictates that all the q components be introduced into the model.

Substituting into equation Eq (3.5), the polynomial has the general usable form:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{13}X_1X_{13} + b_{23}X_2X_3 + b_{11}X^2_1 + b_{22}X^2_2 + b_{33}X^2_3 \quad \ldots\ldots \quad \ldots.. \quad \ldots \quad (3.6)$$

H. Scheffe (1958), suggested to describe the mixture properties by reduced polynomials obtainable from Eqn (3.6) subject to the normalization condition of Eqn. (3.3) for a sum of independent variables. For a ternary mixture, the reduced second degree polynomial can be obtained as follows:

From Eqn. (3.3)

$$X_1 + X_2 + X_3 = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.7)$$

i.e

$$b_0 X_2 + b_0X_2 + b_0 x_3 = b_0 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \ (3.8)$$

Multiplying Eqn. (3.8) by X_1, X_2, x_3, in succession gives

$$\begin{aligned} X_1^2 &= X_1 - X_1X_2 - X_1X_3 \\ X_2^2 &= X_2 - X_1X_2 - X_2X_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \ (3.9) \\ X_3^2 &= X_3 - X_1X_3 - X_2X_3 \end{aligned}$$

substituting Eqn. (3.8) into Eqn. (3.9), we obtain after necessary transformation that

$$\hat{Y} = (b_0 + b_1 + b_{11})X_1 + (b_0 + b_2 + b_{22})X_2 + (b_0 + b_3 + b_{33})X_3 + (b_{12} - b_{11} - b_{22})X_1X_2 + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{23} - b_{22} - b_{33})X_2X_3 \ . \ \ldots \ .. \ \ldots \ (3.10)$$

If we denote

$$\beta_i = b_0 + b_i + b_{ii}$$

and $\beta_{ij} = b_{ij} - b_{ii} - b_{jj}$, then we arrive at the reduced second degree polynomial in 6 variables:

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{23}X_2X_{23} \quad . \; . \quad (3.11)$$

Thus, the number of coefficients has reduced from 10 in Eqn 3.6 to 6 in Eqn 3.11. That is, the reduced second degree polynomial in q variables is

$$\hat{Y} = \sum \beta_iX_i + \sum\beta_{ij}X_i \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3.12)$$

3.2.2 Construction of Experimental/Design Matrix

From the coordinates of points in the simplex lattice, we can obtain the design matrix. We recall that the principal coordinates of the lattice, only a component is 1 (refer to fig 3.1), others are zero.

Hence if we substitute in Eqn. (3.11), the coordinates of the first point (X_1=1, X_2=0, and X_3=0, Table 3.1), we get that $Y_1 = \beta_1$.

And doing so in succession for the other two points if the hexahedron, we obtain

$$Y_2 = \beta_2, Y_3 = \beta_3 \quad .. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.13)$$

The substitution of the coordinates of the fourth point yields

$$Y_{12} = \tfrac{1}{2} X_1 + \tfrac{1}{2}X_2 + \tfrac{1}{2}X_1.X_2$$
$$= \tfrac{1}{2}\beta_1 + \tfrac{1}{2}\beta_2 + {}^1/_4\,\beta_{12}$$

But as $\beta_i = Y_i$ then

$$Y_{12} = \tfrac{1}{2}\beta_1 - \tfrac{1}{2}\beta_2 - {}^1/_4\,\beta_{12}$$

Thus

$$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2 \quad . \quad . \quad . \quad . \quad . \quad (3.14)$$

And similarly,

$$\beta_{13} = 4Y_{13} - 2Y_1 - 2Y_2$$
$$\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3$$

Or generalizing,

$$\beta_i = Y_i \text{and } \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad . \quad . \quad . \quad . \quad . \quad . \quad .(3.15)$$

which are the coefficients of the reduced second degree polynomial for a q-component mixture, since the three points defining the coefficients β_{ij} lie on the edge. The subscripts of the mixture property symbols indicate the relative content of each component X_1 alone and the property of the mixture is denoted by Y_1. Mixture 4 includes X_1 and X_2, and the property being designated Y_{12}.

3.2.3 Actual and Pseudo Components

The requirements of the simplex that

$$\sum_{X=1} Xi = 1$$

Makes it impossible to use the normal mix ratios such as 1:3, 1:5, etc, at a given water/cement ratio. Hence a transformation of the actual components (ingredient proportions) to meet the above criterionis unavoidable. Such transformed ratios say $X_1^{(i)}$, $X_2^{(i)}$, and $X_3^{(i)}$ for the i^th^ experimental points are called pseudo components. Since X_1, X_2 and X_3 are subject to $\sum Xi = 1$, the transformation of cement:laterite:water at say 0.60 water/cement ratio cannot easily be computed because X_1, X_2 and X_3 are in pseudo expressions $X_1^{(i)}$, $X_2^{(i)}$, and $X_3^{(i)}$ .For the i^{th} experimental point, the transformation computations are to be done.

The arbitrary vertices chosen on the triangle are A_1(1:7.50:0.05), A_2(1:8.20:0.03) and A_3(1:6.90:0.10), based on experience and earlier research reports.

3.2.4 Transformation Matrix

If Z denotes the actual matrix of the i^{th} experimental points, observing from Table 3.2 (points 1 to 3),

$$BZ = X = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .(3.16)$$

where B is the transformed matrix.

Therefore, $B = I.Z^{-1}$

Or $B = Z^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.17)$

For instance, for the chosen ratios A_1, A_2 and A_3 (fig. 3.6),

$$Z = \begin{pmatrix} 1.00 & 7.50 & 0.50 \\ 1.00 & 8.20 & 0.30 \\ 1.00 & 6.90 & 0.10 \end{pmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.18)$$

From Eqn 3.17,

$B = Z^{-1}$

$$Z^{-1} = \begin{bmatrix} 26.65 & -17.60 & -8.04 \\ -30.04 & 2.17 & 0.87 \\ -56.52 & 26.08 & 30.43 \end{bmatrix}$$

Hence,

$$B\,Z^{-1} = Z.\,Z^{-1} = 0 \begin{bmatrix} 1 & 0 & 0 \\ & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, for actual component Z, the pseudo component X is given by

$$X \begin{bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \end{bmatrix} = B \begin{bmatrix} 26.65 & -17.60 & -8.04 \\ -30.04 & 2.17 & 0.87 \\ -56.52 & 26.08 & 30.43 \end{bmatrix} Z \begin{bmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \end{bmatrix}$$

which gives the $X_i(i=1,2,3)$ values in Table 3.2.

The inverse transformation from pseudo component to actual component is expressed as

$AX = Z$ . . . . (3.19)

where A = inverse matrix

$A = Z X^{-1}$.

From Eqn 3.16, X = BZ, therefore,

$A = Z.\,(BZ)^{-1}$

$A = Z.Z^{-1}B^{-1}$

$A = IB^{-1}$

$B = B^{-1}$ . . . . (3.20)

This implies that for any pseudo component X, the actual component is given by

$$Z \begin{bmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \end{bmatrix} = B \begin{bmatrix} 1 & 7.50 & 0.05 \\ 1 & 8.20 & 0.03 \\ 1 & 6.90 & 0.10 \end{bmatrix} X \begin{bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \end{bmatrix} \quad . \quad . \quad . \quad . \quad . \qquad (3.21)$$

Eqn 3.21 is used to determine the actual components from points 4 to 6 , and the control values from points 7 to 9 (Table 3.2).

Table 3.2 Values for Experiment

N	X_1	X_2	X_3	RESPONSE	Z_1	Z_2	Z_3
1	1	0	0	Y_1	1	1	1
2	0	1	0	Y_2	7.50	8.20	6.90
3	0	0	1	Y_3	0.05	0.03	0.10
4	1/2	½	0	Y_{12}	1	7.85	0.04
5	½	0	1/2	Y_{13}	1	7.20	0.075
6	0	½	1/2	Y_{23}	1	7.55	0.065
Control Points (6-9)							
7	1/3	1/3	1/3	Y_{123}	1	7.458	0.0594
8	1/3	2/3	0	Y_{122}	1	7.698	0.0366
9	0	1/3	2/3	Y_{233}	1	7.329	0.0769

3.2.5 Use of Values in Experiment

During the laboratory experiment, the actual components were used to measure out the appropriate proportions of the ingredients: cement, laterite and water, for mixing the lateritic concrete materials for casting the samples. The values obtained are presented in Tables in section 5.

3.3 Adequacy of Tests

This is carried out by testing the fit of a second degree polynomial (Akhnarova and Kafarov 1982). After the coefficients of the regression equation has been derived, the statistical analysis is considered necessary, that is, the equation should be tested for goodness of fit, and the equation and surface values bound into the confidence intervals. In experimentation following simplex-lattice designs there are no degrees of freedom to test the equation for adequacy, so, the experiments are run at additional so-called test points.

The number of control points and their coordinates are conditioned by the problem formulation and experiment nature. Besides, the control points are sought so as to improve the model in case of inadequacy. The accuracy of response prediction is dissimilar at different points of the simplex. The variance of the predicted response, S_Y^2, is obtained from the error accumulation law. To illustrate this by the second degree polynomial for a ternary mixture, the following points are assumed:

Xi can be observed without errors (Akhanarova and Kafarov, 1982).

The replication variance, S_Y^2, is similar at all design points, and

Response values are the average of n_i and n_{ij} replicate observations at appropriate points of the simplex

Then the variance $S_{\hat{Y}i}$ and $S_{\hat{Y}ij}$ will $S_{\hat{Y}ij}$ will be

$$(S_{\hat{Y}}^2)_i = S_Y^2/n_i \quad (3.22)$$

$$(S_{\hat{Y}}^2)_{ij} = S_Y^2/n_{ij}. \quad (3.23)$$

In the reduced polynomial,

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_{12}X_{12} + \beta_{13}X_{13} + \beta_{23}X_{23} \quad (3.24)$$

If we replace coefficients by their expressions in terms of responses,

$\beta_i = Y_i$ and $\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$

$$\hat{Y} = Y_1X_1 + \hat{Y}_2X_2 + Y_3X_3 + (4Y_{12} - 2Y_1 - 2Y_2)X_1X_2 + (4Y_{13} - 2Y_1 - 2Y_3)X_1X_3 + (4Y_{23} - 2Y_2 - 2Y_3)X_2X_3$$

$$= Y_1(X_1 - 2X_1X_2 - 2X_1X_3) + Y_2(X_2 - 2X_1X_2 - 2X_2X_3) + Y_3(X_3 - 2X_1X_3 + 2X_2X_3) + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{23}X_2X_3 \quad (3.25)$$

Using the condition $X_1+X_2+X_3 = 1$, we transform the coefficients at Y_i

$$X_1 - 2X_1X_2 - 2X_1X_3 = X_1 - 2X_1(X_2 + X_3) = X_1 - 2X_1(1 - X_1) = X_1(2X_1 - 1) \text{ and so on.} \quad (3.26)$$

Thus

$$\hat{Y} = X_1(2X_1 - 1)Y_1 + X_2(2X_2 - 1)Y_2 + X_3(2X_3 - 1)Y_3 + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{23}X_2X_3 \quad (3.27)$$

Introducing the designation

$$a_i = X_i(2X_1 - 1) \text{ and } a_{ij} = 4X_iX_j \quad (3.27a)$$

and using Eqns (3.22) and (3.230) give the expression for the variance S_Y^2.

$$S_{\hat{Y}}^2 = S_Y^2 \left(\sum_{1\le i\le q} a_{ii}/n_i + \sum_{1\le i<j\le q} a_{jj}/n_{ij}\right) \quad (3.28)$$

If the number of replicate observations at all the points of the design are equal, i.e. $n_i=n_{ij}= n$, then all the relations for $S_{\hat{Y}}^2$ will take the form

$$S_{\hat{Y}}^2 = S_Y^2\xi/n \quad (3.29)$$

where, for the second degree polynomial,

$$\xi = \sum_{1\le i\le q} a_i^2 + \sum_{1\le i<j\le q} a_{ij}^2 \quad (3.30)$$

As in Eqn (3.30), ξ is only dependent on the mixture composition. Given the replication Variance and the number of parallel observations n, the error for the predicted values of the response is readily calculated at any point of the composition-property diagram using an appropriate value of ξ taken from the curve.

Adequacy is tested at each control point, for which purpose the statistic is built:

$$t = \Delta_Y/(S_{\hat{Y}}^2 + S_Y^2) = \Delta_Y n^{1/2}/(S_Y(1 + \xi)^{1/2} \quad (3.31)$$

where $\Delta_Y = Y_{exp} - Y_{theory}$ (3.32)

and n = number of parallel observations at every point.
The t-statistic has the student distribution, and it is compared with the tabulated value of $t_{\alpha/L}(V)$ at a level of significance α, where L = the number of control points, and V = the number for the degrees of freedom for the replication variance.
The null hypothesis is that the equation is adequate is accepted if $t_{cal} < t_{Table}$ for all the control points.
The confidence interval for the response value is

$$\hat{Y} - \Delta \leq Y \leq \hat{Y} + \Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.33)$$

$$\Delta = t_{\alpha/L,k} S_{\hat{Y}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.34)$$

where k is the number of polynomial coefficients determined.
Using Eqn (3.29) in Eqn (3.34)

$$\Delta = t_{\alpha/L,k} S_{Y}(\xi/n)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.35)$$

IV. METHODOLOGY

4.1 Introduction

To be a good structural material, the material should be homogeneous and isotropic. The Portland cement, laterite or concrete are none of these, nevertheless they are popular construction materials (Wilby, 1963).The necessary materials required in the manufacture of the lateritic concrete in the study are cement, laterite and water.

4.1 Materials

The disturbed samples of laterite material were collected at the Vocational Education project site at the University of Nigeria, Nsukka, at the depth of 1.5m below the surface.
The water for use is pure drinking water which is free from any contamination i.e. nil Chloride content, pH =6.9, and Dissolved Solids < 2000ppm. Ordinary Portland cement is the hydraulic binder used in this project and sourced from the Dangote Cement Factory, and assumed to comply with the Standard Institute of Nigeria (NIS) 1974, and kept in an air-tight bag.

4.1.1 Material Properties

All samples of the laterite material conformed to the engineering properties already determined by a team of engineering consultants from the Civil Engineering Department, U.N.N, who reported on the Sieve Analysis Tests, Natural Moisture Content, etc, carried out according to the British Standard Specification, BS 1377 – "Methods of Testing Soils for Civil Engineering Purposes".

4.2 Preparation of Samples

The sourced materials for the experiment were transferred to the laboratory where they were allowed to dry. A samples of the laterite were prepared and tested to obtain the moisture content for use in proportioning the components of the lateritic concrete to be prepared. The laterite was sieved to remove debris and coarse particles. The component materials were mixed at ambient temperature. The materials were mixed by weight according to the specified proportions of the actual components generated in Table 3.2. In all, two blocks of 220mm x210 x120mm for each of six experimental points and three control points were cast for the compressive strength test, cured for 28 days after setting and hardening.

4.3 Strength Test

After 28 day of curing, the cubes and blocks were crushed, with dimensions measured before and at the point of shearing, to determine the lateritic concrete block strength, using the compressive testing machine to the requirements of BS 1881:Part 115 of 1986.

V. RESULT AND ANALYSIS

5.1 Determination of Replication Error And Variance of Response

To raise the experimental design equation models by the lattice theory approach, two replicate experimental observations were conducted for each of the six design points.
Hence we have below, the table of the results (Tables 5.1a,b and c) which contain the results of two repetitions each of the 6 design points plus three Control Points of the (3,2) simplex lattice, and show the mean and variance values per test of the observed response, using the following mean and variance equations below:

$$\ddot{Y} = \sum_{i} (Y_r)/r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5.1$$

where $\hat{Y}$ is the mean of the response values and r =1,2.

$S_Y^2 = \sum[(Y_i - \ddot{Y}_i)^2]/(n-1)$ 5.2
where n = 9.

Table 5.1a Result of the Replication Variance of the Compressive Strength Response for 150x150x150 mm Cube

Experiment No (n)	Repetition	Response E_c (N/mm$^{2)}$	Response Symbol	$\sum Y_r$	$\ddot{Y}_r$	$\sum(Y_r - \ddot{Y}_r)^2$	S_i^2
1	1A 1B	3.25 2.15	Y_1	5.40	2.70	0.605	1380.73
2	2A 2B	3.11 1.94	Y_2	5.05	2.325	0.764	8644.76
3	3A 3B	2.98 1.61	Y_3	4.59	2.295	0.938	1381.48
4	4A 4B	3.42 2.36	Y_{12}	5.78	2.89	2.621	6351.91
5	5A 5B	3.89 1.76	Y_{13}	5.62	2.81	2.205	652.35
6	6A 6B	2.44 2.44	Y23	4.50	2.25	0.072	92452.54
Control Points							
7	7A 7B	2.22 1.51	C_1	3.73	1.865	0.252	3339.91
8	8A 8B	3.01 2.09	C_2	5.10	2.55	0.423	14844.09
9	9A 9B	1.96 2.23	C_3	4.19	2.095	0.036	24296.18

∑3.959

Replication Variance

$S_{Yc}^2 = (\sum S_i^2)/(n-1) = 3.959/8 = 0.494$

Replication Error

$S_{Yc} = (S_{\hat{Y}}^2)^{1/2} = 19293^{1/2} = 0.703$

5.1.2.4 Determination of Regression Equation for the Compressive Strength. From Eqns 3.15 and Table 5.1 the coefficients of the reduced second degree polynomial is determined as follows:

β_1=2.700
β_2= 2.525
β_3=2.295
β_{12}=4(2.89)-2(2.27)-2(5.525) =1.11
β_{13}=4(2.81)-2(2.7)-2(5.595) =1.25
β_{23}=4(2.25)-2(2.525)-2(2.295) = 0.64

Thus, from Eqn (3.11),
$\hat{Y}_c = 2.70X_1 + 2.525X_2 + 2.295X_3 + 1.11X_1X_2 + 1.25X_1X_3 - 0.64X_2X_3$ . . . . . . (5.3a)

Eqn 5.3c is the mathematical model of the G of the lateritic concrete based on the 28-day strength.

5.1.2.5 Test of Adequacy of the Compressive strength Model

Eqn 5.4, the equation model, will be tested for adequacy against the controlled experimental results.

We recall our statistical hypothesis as follows:

1. Null Hypothesis (H_0): There is no significant difference between the experimental values and the theoretical expected results of the compressive strength.
2.Alternative Hypothesis (H_1): There is a significant difference between the experimental values and the theoretical expected results of the compressive strength.

5.1.2.6 t-Test for the Compressive strength Model
If we substitute for X_i in Eqn 5.4 from Table 3.3, the theoretical predictions of the response ($\hat{Y}$) can be obtained. These values can be compared with the experimental results (Table 5.1). For the t-test (Table 5.2), a, ξ, t and Δ_y are evaluated using Eqns 3.25a, 3.28, 3.29 and 3.30 respectively.

Table 5.2 t-Test for the Test Control Points

N	CN	I	J	a_i	a_{ij}	a_i^2	a_{ij}^2	ξ	Ÿ	$\hat{Y}_a$	Δ_y	t
1	C_1	1	2	-0.333	0.444	0.011	0.197		1.865	2.695	0.830	1.028
		1	3	-0.333	0.444	0.011	0.197					
		2	3	-0.333	0.444	0.011	0.197					
					∑	0.033	0.591	0.624				
2	C_2	1	2	-0.333	0.887	0.011	0.787		2.550	2.827	0.277	0.411
		1	3	-0.333	0.000	0.011	0.000					
		2	3	0.333	0.000	0.011	0.000					
					∑	0.033	0.787	0.820				
3	C_3	1	2	0.000	0.000	0.000	0.000		2.095	2.227	0.132.	0.147
		1	3	0.000	0.000	0.000	0.000					
		2	3	-0.333	0.887	0.011	0.787					
					∑	0.011	0.787	0.798				

Significance level α = 0.05,
i.e. $t_{\alpha/L}(V_c) = t_{0.05/3}(9)$, where L=number of control point.

From Appendix A, the tabulated value of $t_{0.05/6}(9)$ is found to be 2.966 which is greater than any of the calculated t-values in Table 5.2. Hence we can accept the Null Hypothesis.
From Eqn 3.35, with k=6 and $t_{\alpha/k}(V) = t_{0.05/6}(9) = 2.966$,
$\Delta = 1.266$ which satisfies the confidence interval equation of
Eqn 3.31 when viewed against the response values in Table 5.2.

5.2 Computer Program
The computer program is developed for the model). In the program any desired Modulus can be specified as an input and the computer processes and prints out possible combinations of mixes that match the property, to the following tolerance:
Compressive Strength - 0.001 N/mm^2,
Interestingly, should there be no matching combination, the computer informs the user of this. It also checks the maximum value obtainable with the model.

5.2.1 Choosing a Combination
It can be observed that the strength of 2.896896 N/sq mm yielded 4 combinations. To accept any particular proportions depends on the factors such as workability, cost and honeycombing of the resultant lateritic concrete.

6.1 Conclusion
Henry Scheffe's simplex design was applied successfully to prove that the modulus of of lateritic concrete is a function of the proportion of the ingredients (cement, laterite and water), but not the quantities of the materials.
The maximum compressive strength obtainable with the compressive strength model is 2.896896 N/sq mm. See the computer run outs which show all the possible lateritic concrete mix options for the desired modulus property, and the choice of any of the mixes is the user's.
One can also draw the conclusion that the maximum values achievable, within the limits of experimental errors, is quite below that obtainable using sand as aggregate. This is due to the predominantly high silt content of laterite.

It can be observed that the task of selecting a particular mix proportion out of many options is not easy, if workability and other demands of the resulting lateritic concrete have to be satisfied. This is an important area for further research work.

The project work is a great advancement in the search for the applicability of laterite in concrete mortar production in regions where sand is extremely scarce with the ubiquity of laterite.

6.2 Recommendations

From the foregoing study, the following could be recommended:

i) The model can be used for the optimization of the strength of concrete made from cement, laterite and water.
ii) Laterite aggregates cannot adequately substitute sharp sand aggregates for heavy construction.
iii) More research work need to be done in order to match the computer recommended mixes with the workability of the resulting concrete.
iii) The accuracy of the model can be improved by taking higher order polynomials of the simplex.

```
'QBASIC BASIC PROGRAM THAT OPTIMIZES THE PROPORTIONS OF LATERITIC CONCRETE
MIXES
'USING THE SCHEFFE'S MODEL FOR CONCRETE COMPRESSIVE STRENGTH
 Cls
C1$ = "(ONUAMAH.HP) RESULT OUTPUT ": C2$ = "A COMPUTER PROGRAM "
C3$ = "ON THE OPTIMIZATION OF THE COMPRESSIVE STRENGTH OF A 3-COMPONENT
LATERITIC CONCRETE MIX"
Print C2$ + C1$ + C3$
Print
'VARIABLES USED ARE
'X1, X2, X3, Z1, Z2, Z3, YT, YTMAX, DS
'INITIALISE I AND YTMAX
I = 0: YTMAX = 0
For MX1 = 0 To 1 Step 0.01
For MX2 = 0 To 1 - MX1 Step 0.01
MX3 = 1 - MX1 - MX2
YTM = 2.70 * MX1 +2.525 * MX2 +2.295 * MX3 +1.11 * MX1 * MX2 +1.25 * MX1 * MX3 – 0.64 * MX2 *
MX3
If YTM >= YTMAX Then YTMAX = YTM
Next MX2
Next MX1
INPUT "ENTER DESIRED  MODULUS, DS = "; DS
'PRINT OUTPUT HEADING
Print
Print Tab(1); "No"; Tab(10); "X1"; Tab(18); "X2"; Tab(26); "X3"; Tab(32); "YTHEORY"; Tab(45); "Z1";
Tab(53); "Z2"; Tab(61); "Z3"
Print
'COMPUTE THEORETICAL  MODULUS, YT
For MX1 = 0 To 1 Step 0.01
For MX2 = 0 To 1 - X1 Step 0.01
MX3 = 1 - X1 - X2
YT =  2.70 * MX1 +2.525 * MX2 +2.295 * MX3 +1.11 * MX1 * MX2 +1.25 * MX1 * MX3 – 0.64 * MX2 *
MX3
If Abs(YT - DS) <=0.001 Then
'PRINT MIX PROPORTION RESULTS
Z1 = X1 + X2 + X3: Z2 = 7.5 * X1 + 8.2 * X2 + 6.9 * X3: Z3 = 0.05 * X1 + 0.03 * X2 + 0.1 * X3
I = I + 1
Print Tab(1); I; USING; "##.###"; Tab(7); X1; Tab(15); X2; Tab(23); X3; Tab(32); YT; Tab(42); Z1; Tab(50);
Z2; Tab(58); Z3
Print
Print
If (X1 = 1) Then GoTo 550
Else
If (X1 < 1) Then GoTo 150
```

```
End If
150     Next X2
Next X1
If I > 0 Then GoTo 550
Print
Print "SORRY, THE DESIRED COMPRESSIVE STRENGTH IS OUT OF RANGE OF MODEL"
GoTo 600
550 Print Tab(5); "THE MAXIMUM VALUE PREDICTABLE BY THE MODEL IS "; YTMAX; "N / Sq mm;
"
600 End
```

A COMPUTER PROGRAM (ONUAMAH.HP) RESULT OUTPUT ON THE OPTIMIZATION OF THE COMPRESSIVE STRENGTH OF A 3-COMPONENT LATERITIC CONCRETE MIX

```
ENTER DESIRED COMPRESSIVE STRENGTH, DS = ?2.2
No      X1      X2      X3    YTHEORY      Z1      Z2      Z3
SORRY, THE DESIRED COMPRESSIVE STRENGTH IS OUT OF RANGE OF MODEL
Press any key to continue
ENTER DESIRED COMPRESSIVE STRENGTH, DS = ? 2.26
No      X1      X2      X3    YTHEORY      Z1      Z2      Z3
 1    0.000   1.000   0.900   2.260      1.000   7.030   0.093
 2    0.000   0.540   0.460   2.260      1.000   7.140   0.087
 3    0.010   0.180   0.810   2.259      1.000   7.140   0.087
 4    0.100   0.460   0.530   2.261      1.000   7.504   0.067

THE MAXIMUM VALUE PREDICTABLE BY THE MODEL IS  2.896896 N / Sq mm;
Press any key to continue
```

A COMPUTER PROGRAM (ONUAMAH.HP) RESULT OUTPUT ON THE OPTIMIZATION OF THE COMPRESSIVE STRENGTH OF A 3-COMPONENT LATERITIC CONCRETE MIX

```
ENTER DESIRED  COMPRESSIVE STRENGTH, DS = ? 2.9
No      X1      X2      X3    YTHEORY      Z1      Z2      Z3
SORRY, THE DESIRED COMPRESSIVE STRENGTH IS OUT OF RANGE OF MODEL
Press any key to continue
```

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